

Exam 3
Chapter 7

Name: _____

Do not write your name on any other page. Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications. Do not approximate answers. Give exact answers. **Only scientific calculators are allowed on this exam.**

Show your work!

1. (10 points) Use Laplace transforms to solve the initial value problem

$$x'' + 3x' + 2x = 2; \quad x(0) = 2, x'(0) = -3.$$

2. (10 points) Use Laplace transforms to solve the initial value problem

$$x'' + 8x' + 25x = 0; \quad x(0) = 2, x'(0) = 3.$$

3. (7 points) Use the fact that $\mathcal{L}\{(f * g)(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$ to find

$$\mathcal{L}^{-1}\left\{\frac{98}{(s-2)(s-3)}\right\}.$$

4. (7 points) Use the fact that $f(t) = -\frac{1}{t}\mathcal{L}^{-1}\{F'(s)\}$ to find

$$\mathcal{L}^{-1}\left\{\ln\left(1 + \frac{1}{s^2}\right)\right\}.$$

5. (7 points) Use the fact that $\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s)$ to find

$$\mathcal{L}\{f(t)\} \quad \text{where } f(t) = \begin{cases} \cos \pi t, & \text{if } 0 \leq t \leq 2 \\ 0, & \text{if } t > 2. \end{cases}$$

6. Consider the differential equation

$$y^{(4)} + 2y'' + y = 4t^3e^t; \quad y(0) = y'(0) = y''(0) = y^{(3)}(0) = 0.$$

- (a) (4 points) Solve for the transform $Y(s) = \mathcal{L}\{y(t)\}$.
(Hint: You may need the formula $\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$ or any other method.)
- (b) (5 points) Find the general form of the partial fraction decomposition of $Y(s)$. You do not need to solve for the coefficients.